A POSTERIORI ERROR ESTIMATES BASED ON POLYNOMIAL PRESERVING GRADIENT RECOVERY

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A recovery type *a posteriori* error estimator is proposed. The estimator is based on a polynomial preserving gradient recovery (PPR) and can be used for arbitrary triangulations. The method maintains the simplicity, efficiency, and superconvergence property of the Zienkiewicz-Zhu patch recovery (SPR) [3,4]. The difference is that at each node, SPR fits (in a discrete least-squares sense) a vector polynomial with gradient values of the numerical solution at some local sampling points, while PPR fits a polynomial with function values of the numerical solution at some local nodal points and then takes gradient.

The error estimator based on SPR recovery has been demonstrated to be one of the most robust error estimators [1,2]. By testing on a set of benchmark problems [6], the *a posteriori* error estimator based on PPR-recovered gradient was found to be as good as or better than the estimator based on the SPR-recovered gradient. Further analysis has revealed that the PPR has better superconvergence property from a theoretical point of view. In fact, PPR is superconvergent for the Chevron triangulation and ultra-convergent (i.e., two orders higher than the optimal global rate) at element edge centers for the regular triangulation [5], a property not shared by SPR.

References

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